We used the property that for any real number x, $x^0 = 1$.

Recall that the derivative of $\log_e x$ is $\frac{1}{x}$. Then the anti derivative of $\frac{1}{x}$ is $\log_e x$. Notice that $\frac{1}{x} = x^{-1}$, and that if we had used the rules we have developed to find the anti derivatives of things like x^m, we would have the anti derivative of x^{-1} being $\frac{x^{-1+1}}{-1+1} = \frac{x^0}{0}$ which is not defined as we can not divide by zero. So we have the special rule for the anti derivative of $1/x$:

$$\int \frac{1}{x}\,dx = \log_e x + c$$

Recall that the derivative of $\log_e f(x)$ is $\frac{f'(x)}{f(x)}$. Then we have

$$\int \frac{f'(x)}{f(x)}\,dx = \log_e f(x) + c$$

<u>Example 3</u> : Evaluate the indefinite integral $\int \frac{5}{5x+2}\,dx$. This has the form $\int \frac{f'(x)}{f(x)}\,dx$ so we get

$$\int \frac{5}{5x+2}\,dx = \log_e(5x+2) + c$$

Note that when you need to integrate a function like $1/(ax+b)$ (where a and b are constants), then

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\int \frac{a}{ax+b}\,dx = \frac{1}{a}\log_e(ax+b) + c$$

<u>Example 4</u> : Find the area under the curve $f(x) = 1/(2x+3)$ between 3 and 11.

$$\begin{aligned} A &= \int_3^{11} \frac{1}{2x+3}\,dx \\ &= \frac{1}{2}\log_e(2x+3)\Big]_3^{11} \\ &= \frac{1}{2}\log_e(2\times 11+3) - \frac{1}{2}\log_e(2\times 3+3) \\ &= \frac{1}{2}\log_e 25 - \frac{1}{2}\log_e 9 \\ &= \log_e(25)^{\frac{1}{2}} - \log_e(9)^{\frac{1}{2}} \\ &= \log_e\frac{5}{3} \end{aligned}$$